

Long-range Self-interacting Dark Matter in the Sun

Jing Chen,^{1,2,*} Zheng-Liang Liang,^{3,†} Yue-Liang Wu,^{2,‡} and Yu-Feng Zhou^{2,§}

¹*University of Chinese Academy of Science*

19A Yuquan Road, Beijing, 100049, P.R. China

²*State Key Laboratory of Theoretical Physics,*

Kavli Institute for Theoretical Physics China,

Institute of Theoretical Physics, Chinese Academy of Science

Zhong Guan Cun Street 55#, Beijing, 100190, P.R. China

³*Institute of High Energy Physics, Chinese Academy of Science*

19B Yuquan Road, Beijing, 100049, P.R. China

Abstract

We investigate the implications of the long-rang self-interaction on both the self-capture and the annihilation of the self-interacting dark matter (SIDM) trapped in the Sun. Our discussion is based on a specific SIDM model in which DM particles self-interact via a light scalar mediator, or Yukawa potential, in the context of quantum mechanics. Within this framework, we calculate the self-capture rate across a broad region of parameter space. While the self-capture rate can be obtained separately in the Born regime with perturbative method, and in the classical limits with the Rutherford formula, our calculation covers the gap between in a non-perturbative fashion. Besides, the phenomenology of both the Sommerfeld-enhanced s - and p -wave annihilation of the solar SIDM is also involved in our discussion. Moreover, by combining the analysis of the Super-Kamiokande (SK) data and the observed DM relic density, we constrain the nuclear capture rate of the DM particles in the presence of the dark Yukawa potential. The consequence of the long-range dark force on probing the solar SIDM turns out to be significant if the force-carrier is much lighter than the DM particle, and a quantitative analysis is provided.

*Electronic address: jchen@itp.ac.cn

†Electronic address: liangzl@itp.ac.cn

‡Electronic address: ylwu@itp.ac.cn

§Electronic address: yfzhou@itp.ac.cn

I. INTRODUCTION

Although the existence of dark matter (DM) have been well established by the observations from the galactic scale up to the cosmological scale, the particle nature of DM still remains unclear. While the collisionless Weakly Interacting Massive Particle (WIMP) has been the focus of the current DM study, the possibility of the self-interacting dark matter (SIDM) has also drawn increasing attention in both particle physics and astrophysics (see e.g. [1–6]). Long-range self-interactions between DM particles induced by the exchange of light mediator particles can lead to the Sommerfeld enhancement of DM annihilation cross sections [7–12], which can be used to explain the observed cosmic-ray positron excesses [13–16] by providing a source of the boost factor of $\mathcal{O}(10^2 - 10^3)$ (for a recent global analysis on the boost factor, see e.g. [17–23]). It is under active investigation whether an excess of antiproton also exists in the AMS-02 \bar{p}/p data [24–31] and a boost factor is also required in the antiproton channel. Phenomenological consequences of the light mediator exchange between DM particles in astrophysics and cosmology have been discussed in a series of studies [10–12, 32–49] which include its implications on relic density [11, 12, 36, 37], CMB [37–39], halo shape [35, 42, 50], and small scale structures [32, 46, 51], etc.

Another interesting phenomenology associated with the DM self-interaction is its self-capture within the Sun. In addition to solar elements, DM particles that have already been captured within the Sun may also contribute to the capture of the passing halo ones. Consequently, more DM particles will build up within the Sun and an enhancement of the high-energy neutrino signals from the DM annihilation might be possible. Under the assumption of a constant self-scattering cross section, the authors of Ref. [52–54] utilize neutrino detectors to probe the solar SIDM particles and explore for the viable parameter space. It is found that a significant enhancement of the solar neutrino flux requires a disparity between two separate input parameters, *i.e.*, a large DM self-capture rate and a small annihilation cross section. In this paper, we will extend this case to the scenario where the DM particles interact with each other via a light mediator (also in Ref. [55, 56]*). In such scenario, this force-carrier will play a role in both the self-capture and annihilation within the Sun, as well as in thermal freeze-out that accounts for the observed DM density. As a result, the self-capture and annihilation rate become inter-related to each other, and more than that, the disparity between them may be achieved in a natural way. To investigate the relevant implications from an optimistic view, we examine a concrete SIDM model in which the Majorana DM particles χ interact with each other through the exchange of a light scalar mediator ϕ , so that the “irreducible” annihilation process $\chi\chi \rightarrow \phi\phi$ proceeds through the p -wave channel. The p -wave annihilation is of particular interest because contrary to its s -wave counterpart that is subject to the Sommerfeld enhancement, it suffers a velocity-suppression not only in the Sun but at the kinetic decoupling stage, and hence results in a larger relic density [12]. As a consequence, a larger self-interaction coupling α_χ is required accordingly for the p -wave annihilation to give the correct relic density, which is also a prerequisite for

* In these two papers, the self-capture rate is calculated either in the Coulomb limit [55] or with the perturbative method [56].

a large self-capture rate.

Following the literature [11, 35, 42] in which authors managed to discuss the DM self-scattering beyond the perturbative level, we similarly give a quantitative description of the self-capture rate for the Majorana DM particles. However, instead of adopting the transfer cross section as a proxy quantity to describe the self-scattering, we calculate the self-capture rate with long-range interaction in a straightforward manner, considering that the kinetic requirement for self-capture automatically regulates the forward-scattering divergence and hence makes the self-capture rate a well-defined quantity.

In addition, in order to explore the phenomenology of the solar SIDM from a comprehensive perspective, our discussion includes the Sommerfeld-enhanced annihilation of the solar SIDM as well. We illustrate that while the DM particles predominantly annihilate to light mediators through the p -wave channel, a small but non-vanishing quota of annihilation proceeding through the s -wave channel in the thermal freeze-out may become dominant over its p -wave counterpart within the Sun, and produce signals at the terrestrial neutrino telescopes. For such case we use the neutrino telescope Super-Kamiokande [57] to find out whether the Sommerfeld effect can help constrain the nuclear capture rate C_n in a nontrivial manner. These constraints can be translated into upper limits on the DM-nucleon cross section for other SIDM models. It is found that in the approach of the Coulomb limit of the long-range self-interaction, the constraints from the Super-Kamiokande on nuclear capture rate can be significantly tightened.

This paper is organized as follows. In Sec. II we take an introduction to the model setup for our study. In Sec. III we introduce the related background about the solar DM, including its self-capture and evolution within the Sun. We discuss the implications of the light mediator on the self-capture and annihilation of the solar DM particles, as well as on relic density in Sec. IV. We further investigate its impact on the neutrino telescope Super-Kamiokande in Sec. V, and conclusions are drawn in Sec. VI.

II. MODEL SETUP

We are interested in the the dark sector that gives rise to the Yukawa potential of the DM self-interaction. As a familiar prototype, Yukawa potential between DM particles provides us with a representative description and a concrete example of the long-range self-interaction [10, 12, 42]. The self-interacting Yukawa potential* is written as

$$V(r) = \pm \frac{\alpha_\chi}{r} e^{-m_\phi r}, \quad (1)$$

which describes in non-relativistic limit the following interactions of the dark sector [42]:

$$\mathcal{L}_{\chi\phi} = \begin{cases} g_\chi \bar{\chi} \gamma^\mu \chi \phi_\mu & \text{vector mediator} \\ g_\chi \bar{\chi} \chi \phi & \text{scalar mediator} \end{cases}, \quad (2)$$

* Strictly speaking, Yukawa potential narrowly refers to the attractive interaction in Eq. (1). However, for the purpose of convenience, here we use it to refer to both the attractive and repulsive interactions.

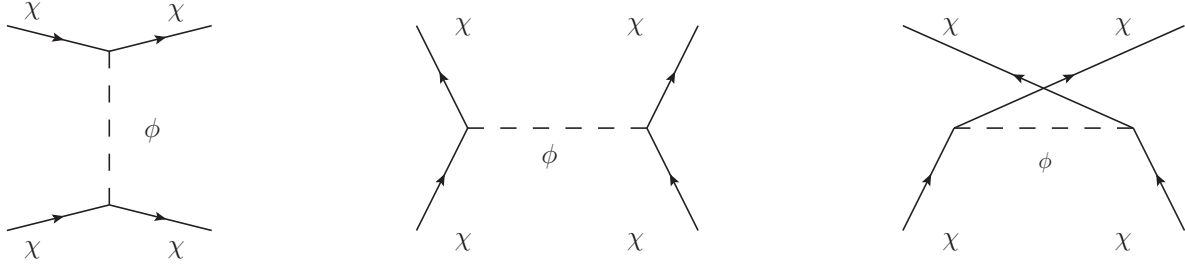


Figure 1: The tree-level diagrams for elastic scattering process $\chi\chi \rightarrow \chi\chi$

where m_ϕ is the mass of the mediator particle ϕ , and the dark fine-structure constant α_χ is connected to the coupling g_χ through $\alpha_\chi = g_\chi^2/(4\pi)$. For a vector force-carrier, the interaction between DM could be attractive ($\bar{\chi}\chi$ scattering) or repulsive ($\chi\chi$ or $\bar{\chi}\bar{\chi}$ scattering), while the force can only be attractive (–) between DM for the scalar mediator scenario. Since in this paper we are interested in spin-1/2 identical DM, i.e., Majorana DM particles, the interaction between DM and the vector mediator in Eq. (2) is absent, and an extra s -channel process should be included in the calculation of relevant scattering amplitude for a scalar mediator ϕ , which is shown in Fig. 1. While the second and the third diagrams in Fig. 1 correspond to the long-range Yukawa interaction in non-relativistic limit, the first s -channel process can be described non-relativistically with a contact interaction, which is subject to the Sommerfeld enhancement that will be reviewed in detail below. However, due to its highly suppressed cross section $\sigma_{\chi\chi} \sim \alpha_\chi^2 v_{\text{rel}}^3/m_\chi^2$, contribution of the s -channel process can be safely ignored compared with that of the Yukawa potential, even if the Sommerfeld enhancement is involved. So only the Yukawa potential is relevant for the DM scattering.

Although we want to explore phenomenologically the impact of the DM long-range self-interaction as far as possible, it is unrealistic for our discussion to be entirely model-independent. At the very least, the phenomenological consequence of the irreducible annihilation process $\chi\chi \rightarrow \phi\phi$ has to be taken into consideration (see Fig. 2). So, we study a model that not only gives rise to significant Sommerfeld effects within the Sun, but also cover as wide a range of phenomenology as possible, namely, the self-capture and both the s - and p -wave annihilation of the solar SIDM.

To facilitate the novelty brought by the long range self-interaction, we assume that the irreducible p -wave process (shown in Fig. 2) dominates the annihilation during the thermal freeze-out. The reason for this assumption is two-fold: 1) As opposed to the s -wave-dominated case, the p -wave-dominated annihilation avoids severe constraint on the coupling α_χ imposed by the DM relic density, so that the Sommerfeld effect remains significant [12, 58]. 2) The p -wave-dominated annihilation within the Sun is subject to such significant velocity-suppression that the total annihilation rate Γ_A will be enhanced remarkably (see Eq. (9) and (18) in the following section), which may further favor the indirect detection of the solar DM.

On the other hand, given the dark sector of the Majorana DM χ and the scalar mediator ϕ , it is natural to assume that they couple to the SM sector through the Higgs portal [59–65]. In addition, extra four-fermion interactions are introduced to account for the DM s -wave

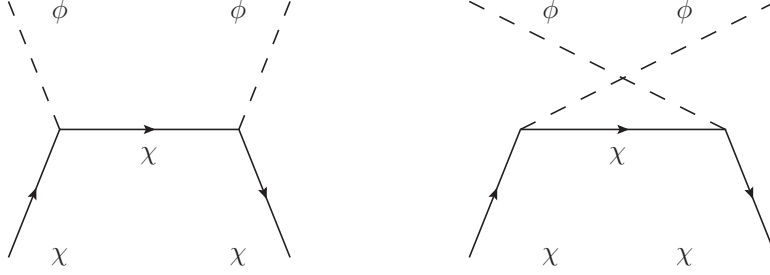


Figure 2: The diagrams for the minimal annihilation process $\chi\chi \rightarrow \phi\phi$.

annihilation into the SM particles. Integrating all these considerations, we consider the following interaction as an example:

$$\mathcal{L} \supset \frac{g_\chi}{2} \bar{\chi}\chi\phi - \sum_f y_f \sin\theta \phi \bar{f}f + y_N \phi N^c N + \sum_f \frac{G_f}{2} i \bar{\chi} \gamma^5 \chi \bar{f}f, \quad (3)$$

where $\sin\theta$ describes the mixing between ϕ and Higgs boson, and $y_f = m_f/v_{\text{EW}}$ is the Yukawa coupling of the SM fermion f , with the SM fermion mass m_f and the Higgs vacuum expectation value $v_{\text{EW}} \approx 246 \text{ GeV}$. G_f is the coupling strength between the DM and the SM fermion f . The coupling of ϕ and right-handed sterile neutrino N is introduced in Ref. [66] to save the light mediator ϕ from the Big Bang Nucleosynthesis (BBN) constraints. To avoid the overclosure problem, overproduction of entropy and changes to the light element abundances during the BBN, ϕ is required to decay before the BBN ($t \sim 1 \text{ s}$) [67], which corresponds to the bound [66]

$$y_N \gtrsim 6 \times 10^{-12} \left(\frac{100 \text{ MeV}}{m_\phi} \right)^{1/2}. \quad (4)$$

III. DARK MATTER IN THE SUN

A. Dark matter in the Sun

The accumulation of the DM in the sun is initiated by DM-nuclei collisions that cost the passing-by Galactic DM particles sufficient kinetic energy to escape from the gravitational pull of the Sun. In this process, subsequent collisions tend to further lower the kinetic energy of the DM rather than eject them back into the deep space as long as the DM mass is heavier than a few GeV, hence the number of the trapped DM grows until their capture and annihilation reach an equilibrium. If the DM self-interaction is taken into account, the evolution of the solar DM number N_χ is depicted by the following equation

$$\frac{dN_\chi}{dt} = C_n + (C_s - E_n) N_\chi - (C_a + E_s) N_\chi^2, \quad (5)$$

where C_n is the DM capture rate by (scattering off) nuclei in the Sun, C_a is twice the annihilation rate of a pair of DM particles, E_n and E_s represent respectively the evaporation

rate by nuclei and by the captured DM particles, and C_s is the capture rate by the trapped DM particles, a key physical quantity of our concern in this paper. It should be noted that in the range of the DM mass of our interest ($m_\chi \geq 10 \text{ GeV}$) it is justified for us to omit the the evaporation rate E_n in Eq. (5), since it is irrelevant for a DM mass larger than 5 GeV [68–70].* Besides, we also omit the the self-evaporation rate E_s because $C_a \gg E_s$, which will be demonstrated in detail in Appendix B. Thus Eq. (5) is reduced to

$$\frac{dN_\chi}{dt} = C_n + C_s N_\chi - C_a N_\chi^2, \quad (6)$$

which has an analytic solution

$$N_\chi = \frac{C_n \tanh(t \cdot \xi)}{\xi - (C_s/2) \tanh(t \cdot \xi)}, \quad (7)$$

with

$$\xi = \sqrt{C_n \cdot C_a + C_s^2/4}. \quad (8)$$

If $\tanh(t_\odot \cdot \xi) \simeq 1$ in Eq. (7) with solar age $t_\odot = 4.5 \times 10^9 \text{ yr}$, the capture and annihilation of the DM reach an equilibrium today. Then the annihilation rate Γ_A for a steady number of DM within the Sun can be written as

$$\begin{aligned} \Gamma_A &= \frac{1}{2} C_a N_\chi^2 \\ &= \frac{1}{2} C_a \left(\sqrt{\frac{C_n}{C_a} + \frac{C_s^2}{4C_a^2}} + \frac{C_s}{2C_a} \right)^2 \\ &= \frac{1}{2} \left(C_n + \frac{C_s^2}{2C_a} + \sqrt{\frac{C_n}{C_a} + \frac{C_s^2}{4C_a^2}} \cdot C_s \right), \end{aligned} \quad (9)$$

which is reduced to the trivial case where the dark matter self-interaction is absent, i.e., when $\Gamma_A = C_n/2$ if $C_s^2/(C_n C_a) \ll 1$ is satisfied.

B. Self-capture and annihilation of the self-interacting dark matter

The formula for the self-capture rate can be obtained after a minor modification to the procedures in Ref. [69, 72] as the following:

$$C_s N_\chi = \frac{\rho_\chi}{m_\chi} \int_{\text{Sun}} n_\chi(r) d^3r \int \sigma_{\text{sc}}(w) \frac{w^2}{u} f(\mathbf{u}) d^3u, \quad (10)$$

where $\rho_\chi = 0.3 \text{ GeV}$ is the DM local density in our solar neighborhood, m_χ manifestly the mass of the DM. $f(\mathbf{u})$ is the velocity distribution of DM in the Sun's rest frame, which is expressed as

$$f(\mathbf{u}) = \frac{e^{-\frac{(\mathbf{u}+\mathbf{v}_\odot)^2}{v_0^2}}}{N(v_{\text{esc}})}, \quad (11)$$

* For smaller DM masses, the detection of the solar DM evaporation is discussed in Ref. [71].

where $|\mathbf{v}_\odot| = 220 \text{ km} \cdot \text{s}^{-1}$ is the velocity of the Sun, \mathbf{u} being the DM velocity at infinity where the solar gravitational effects is negligible, $v_0 = 220 \text{ km} \cdot \text{s}^{-1}$ the DM velocity dispersion, and $N(v_{\text{esc}})$ is the normalization constant dependent on the Galactic escape velocity $v_{\text{esc}} = 544 \text{ km} \cdot \text{s}^{-1}$. $w = \sqrt{v_{\text{esc}}^2(r) + u^2}$ is the velocity of the incident DM particle, with $v_{\text{esc}}(r)$ the solar escape velocity at radius r . The DM distribution within the Sun can be described with a characteristic temperature T_χ as follows*

$$n_\chi(r) \propto \exp \left[-\frac{m_\chi}{T_\chi} \int_0^r \frac{GM(r')}{r'^2} dr' \right], \quad (12)$$

where G is the Newton's constant and $M(r)$ represents the solar mass contained within radius r . T_χ is used to be determined self-consistently through the following equation:

$$T_\chi = T_\odot(R_\chi), \quad (13)$$

in which the right hand side represents the solar temperature at the mean value of DM radius

$$R_\chi = \frac{\int_{\text{Sun}} n_\chi(r) r d^3r}{\int_{\text{Sun}} n_\chi(r) d^3r}. \quad (14)$$

We use the Standard Sun Model (SSM) GS98 [73] to obtain a fitting function of R_χ as follows

$$R_\chi \simeq 0.012 \times \sqrt{\frac{100 \text{ GeV}}{m_\chi}} R_\odot, \quad (15)$$

with solar radius R_\odot . The effective cross section for self-capture, $\sigma_{\text{sc}}(w)$, is dependent on the velocity of the incoming DM particle w , if we approximate the target DM trapped within the Sun to be at rest compared with the accelerated Galactic ones. On the other hand, for the mass range concerned in this paper ($m_\chi \geq 10 \text{ GeV}$), we further assume that the captured DM particles are located at the center of the Sun (see Eq. (15)), thus the self-capture rate can be simplified as

$$C_s = \frac{\rho_\chi}{m_\chi} \int \sigma_{\text{sc}}(w) \frac{w^2}{u} f(\mathbf{u}) d^3u. \quad (16)$$

where $w = \sqrt{v_c^2 + u^2} \equiv \sqrt{v_{\text{esc}}^2(0) + u^2}$.

However, here we note that instead of growing infinitely with the accretion of the captured DM particles, $C_s N_\chi$ saturates to a maximum value set by the geometric limit of the resident DM, above which $C_s N_\chi$ is replaced by

$$\begin{aligned} (C_s N_\chi)_{\text{max}} &= \frac{\rho_\chi}{m_\chi} \pi R_\chi^2 \int \left(\frac{v_{\text{esc}}^2(R_\chi) + u^2}{u} \right) f(\mathbf{u}) d^3u \\ &\simeq \frac{\rho_\chi}{m_\chi} \pi R_\chi^2 \int \left(\frac{v_c^2 + u^2}{u} \right) f(\mathbf{u}) d^3u \\ &\simeq 4.67 \times 10^{24} \times \left(\frac{100 \text{ GeV}}{m_\chi} \right)^2 \cdot \text{s}^{-1}. \end{aligned} \quad (17)$$

* To avoid distraction we postpone a discussion on the solar SIDM distribution to Appendix A.

On the other hand, the annihilation rate of the solar DM can be expressed as

$$C_a = \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\odot} \frac{\int_{\text{Sun}} n_{\chi}^2(r) d^3r}{\left(\int_{\text{Sun}} n_{\chi}(r) d^3r \right)^2},$$

$$\equiv \frac{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\odot}}{V_{\text{eff}}}, \quad (18)$$

where $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\odot}$ is the thermal average of the DM annihilation cross section times relative velocity v_{rel} within the Sun, and the effective volume occupied by the trapped DM particles can also be described with a fitting function as follows:

$$V_{\text{eff}} = 6.9 \times 10^{27} \left(\frac{100 \text{ GeV}}{m_{\chi}} \right)^{3/2} \text{ cm}^3. \quad (19)$$

IV. IMPLICATIONS OF THE LONG-RANGE SELF-INTERACTING DM

In this section, we study the implications of the long-range self-interaction on the self-capture and annihilation within the Sun. Due to the multiple exchanges of the light force-carrier between DM particles, the relevant calculations have to be carried out beyond the perturbative approach. The quantum mechanical description of the two-body scattering is proved to be an effective framework to do so, which is connected to the self-capture and annihilation in a manner that the asymptotic wave-function at infinity describes the final state of the scattering, and the wave-function at origin partly determines the probability of the annihilation. Now we delve into the details.

A. Calculation of the self-capture rate

We calculate the self-capture cross section $\sigma_{\text{sc}}(w)$ in Eq. (10) in the center-of-mass (CM) frame, in which the self-capture differential cross section for a pair of identical DM can be expressed as (after averaging over the initial spins and summing over the final spins)

$$\frac{d\sigma_{\text{sc}}}{d\Omega} = \frac{1}{2} \left[\sum_S (2S+1) \right]^{-1} \cdot \left(\sum_S \left| f(\theta) + (-1)^S \cdot f(\pi - \theta) \right|^2 \right), \quad (20)$$

where S represents the total spin of the DM-pair, the factor $1/2$ accounting for the symmetry of the out-going DM particles, and the elastic scattering amplitude $f(\theta)$ is related to the wave function in position space via

$$\langle \mathbf{x} | \psi_{\mathbf{p}}^+ \rangle = \frac{1}{(2\pi)^{3/2}} \sum_{\ell} i^{\ell} (2\ell+1) A_{\ell}(r) P_{\ell}(\cos \theta)$$

$$\xrightarrow{\text{large } r} \frac{1}{(2\pi)^{3/2}} \left[e^{i\mathbf{p} \cdot \mathbf{x}} + f(\theta) \frac{e^{ipr}}{r} \right], \quad (21)$$

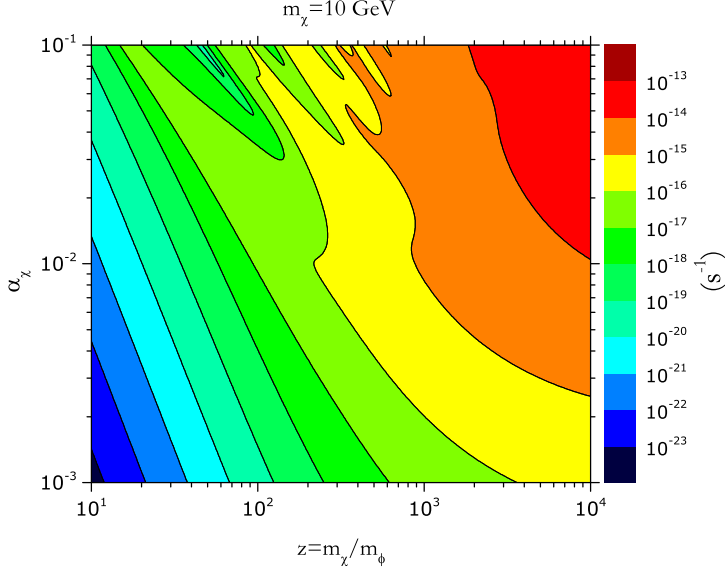


Figure 3: Shown is the self-capture rate C_s dependent on parameter $z = m_\chi/m_\phi$ and α_χ for the Majorana DM with mass $m_\chi = 10 \text{ GeV}$.

where the radial partial-wave amplitude $A_\ell(r)$ has the following asymptotic solution for a large r :

$$A_\ell(r) = e^{i\delta_\ell} [\cos \delta_\ell j_\ell(pr) - \sin \delta_\ell n_\ell(pr)], \quad (22)$$

with $j_\ell(n_\ell)$ as the spherical Bessel (Neumann) function, and δ_ℓ the phase shift for a partial wave ℓ . Thus $f(\theta)$ can be further expressed as

$$f(\theta) = \frac{1}{p} \sum_{\ell=0}^{+\infty} (2\ell+1) e^{i\delta_\ell} \sin \delta_\ell P_\ell(\cos \theta), \quad (23)$$

with $p = m_\chi w/2$. As for the scattering angle θ , to ensure a net capture it is required to satisfy

$$-c \leq \cos \theta \leq c, \quad (24)$$

with

$$c = \frac{v_c^2 - u^2 - 2v_J^2}{v_c^2 + u^2}, \quad (25)$$

where $v_J = 18.5 \text{ km} \cdot \text{s}^{-1}$ is the solar escape velocity at the radius of Jupiter's orbit. As a conservative estimate [74], v_J is introduced not only to take into account the gravitational effects of the Jupiter from a more realistic three-body perspective, but to regulate the divergence encountered in the calculation of C_s for the long-range self-interaction scenario.

We follow the same procedure as proposed in Ref. [42] to numerically calculate the phase shift δ_ℓ and then the self-capture rate C_s for specific parameters m_χ , m_ϕ , and coupling strength α_χ . The value of C_s is scanned in Fig. 3 over the parameter space of the mass ratio $z = m_\chi/m_\phi$ and the self-interaction strength α_χ , for the Majorana DM with mass $m_\chi = 10 \text{ GeV}$. While the Born regime occupies the lower left corner of the figure, the

Sommerfeld effect becomes increasingly significant in the Coulomb limit ($z \rightarrow \infty$), where a large partial wave number ($\ell_{\max} \gtrsim 1000$) is required in calculation to ensure the convergence of C_s . It is worth mentioning that given z and α_χ fixed, C_s will be suppressed for larger DM masses through a m_χ^{-3} dependence, which is obvious from Eq. (16, 20, and 23).

B. Sommerfeld enhancement of the annihilation rate

Apart from the self-capture, the light force-carrier brings about significant consequences on the annihilation of DM as well, which have been widely studied in determining the thermal relic density and in calculating signals in indirect DM searches. To gauge the Sommerfeld effect in annihilation we invoke the Sommerfeld factor as follows [75, 76]

$$\mathcal{S}_\ell = \frac{\text{non-perturbative partial wave cross section}}{\text{tree-level partial wave cross section}} = \left| \frac{\mathcal{M}_{\text{ladder}}^\ell}{\mathcal{M}_0^\ell} \right|^2, \quad (26)$$

where ℓ stands for the ℓ -th partial wave. If one skips somewhat formal theoretical discussions, the implication of Sommerfeld enhancement on annihilation is encoded in the following heuristic relation between the non-perturbative $\mathcal{M}_{\text{ladder}}^\ell$ and “bare” amplitude \mathcal{M}_0^ℓ at tree-level [75, 76]:

$$\mathcal{M}_{\text{ladder}}^\ell = \int \mathcal{M}_0^\ell(\{\mathbf{k}, -\mathbf{k}\} \rightarrow \{\mathbf{p}_f\}) \phi(\mathbf{k}) d^3k, \quad (27)$$

where $\phi(\mathbf{k}) = [\langle \mathbf{k} | \psi_{\mathbf{p}}^+ \rangle + (-1)^S \langle -\mathbf{k} | \psi_{\mathbf{p}}^+ \rangle] / 2$ is the familiar scattering wave function for identical DM-pair in the reduced system, $\mathbf{p} = m_\chi \mathbf{v}_{\text{rel}} / 2$ being the initial momentum in the CM frame, with relative velocity \mathbf{v}_{rel} . It is noted that for identical fermion pairs $L + S$ must be even, thus we have $\phi(\mathbf{k}) = \langle \mathbf{k} | \psi_{\mathbf{p}}^+ \rangle$. After Fourier transformation and by repeated use of addition theorem the Sommerfeld factor for partial wave ℓ can be written in terms of the wave function in position space

$$\mathcal{S}_\ell = \left| \frac{(2\ell + 1)!!}{p^\ell \ell!} \frac{\partial^\ell A_\ell(r)}{\partial r^\ell} \right|_{r=0}^2, \quad (28)$$

which can be calculated numerically along with δ_ℓ . Therefore, after taking into consideration the Sommerfeld enhancement, we express the annihilation rate for the trapped DM particles as follows

$$\begin{aligned} C_a &= \frac{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_\odot}{V_{\text{eff}}} \\ &= \frac{1}{V_{\text{eff}}} \int (\sigma_{\text{ann}} |\mathbf{v}_2 - \mathbf{v}_1|)_{\text{tree}} \mathcal{S} f_\chi(\mathbf{v}_1) f_\chi(\mathbf{v}_2) d^3v_1 d^3v_2 \\ &= \frac{1}{V_{\text{eff}}} \int (\sigma_{\text{ann}} v_{\text{rel}})_{\text{tree}} \mathcal{S} f_\chi(\mathbf{v}_{\text{rel}} / \sqrt{2}) d^3(v_{\text{rel}} / \sqrt{2}), \end{aligned} \quad (29)$$

where \mathcal{S} is the Sommerfeld factor dependent on v_{rel} , the footnote “tree” indicating that the cross section is calculated at the tree-level, and the Maxwellian velocity distribution is

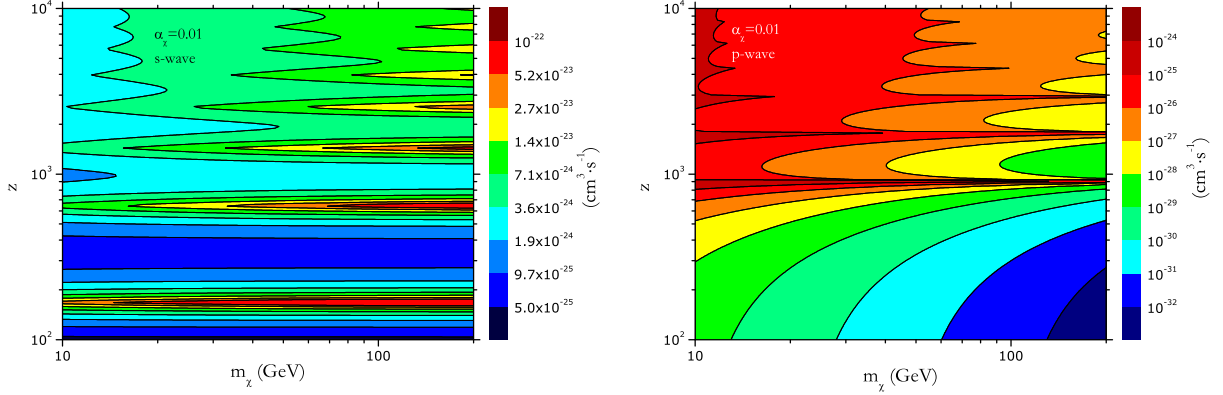


Figure 4: Shown are the thermally averaged cross section $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\odot}$ with $\alpha_{\chi} = 0.01$, for the s -wave and p -wave annihilation within the Sun, respectively. See text for details.

written as

$$f_{\chi}(\mathbf{v}) = \frac{e^{-v^2/u_0^2}}{(\pi u_0^2)^{3/2}}, \quad (30)$$

with $u_0 = \sqrt{2T_{\chi}/m_{\chi}}$. Note that in Eq. (30) we neglect the cut-off at the solar escape velocity, which is equivalent to the approximation $v_{\text{esc}} \rightarrow \infty$.

Since we are only interested in the case where other p -wave processes are suppressed by couplings $y_f \sin \theta$ or y_N in Eq. (3), the total thermally averaged cross section can be written as

$$\begin{aligned} \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle &= \langle (\sigma_{\phi\phi} v_{\text{rel}})_{\text{tree}} \mathcal{S}_1 \rangle + \left\langle \sum_f (\sigma_{\bar{f}f} v_{\text{rel}})_{\text{tree}} \mathcal{S}_0 \right\rangle \\ &= \langle (\sigma_{\phi\phi} v_{\text{rel}})_{\text{tree}} \mathcal{S}_1 \rangle + \langle \eta (\sigma v_{\text{rel}})_0 \mathcal{S}_0 \rangle, \end{aligned} \quad (31)$$

where included are the thermal cross sections for the irreducible p -wave process

$$(\sigma_{\phi\phi} v_{\text{rel}})_{\text{tree}} = \frac{3\pi\alpha_{\chi}^2 v_{\text{rel}}^2}{8m_{\chi}^2}, \quad (32)$$

and its s -wave counterparts arising from the four-fermion interaction

$$(\sigma_{\bar{f}f} v_{\text{rel}})_{\text{tree}} = \frac{G_f^2}{2\pi} c_f m_{\chi}^2 \left(1 - \frac{m_f^2}{m_{\chi}^2} \right)^{3/2}, \quad (33)$$

with the color factor $c_f = 3$ (1) for quarks (leptons). \mathcal{S}_1 (\mathcal{S}_0) is the p -wave (s -wave) Sommerfeld factor. The s -wave component is parametrized as a small factor η times the typical averaged thermal cross section $(\sigma v_{\text{rel}})_0 = 3 \times 10^{-26} \text{ cm}^3 \cdot \text{s}^{-1}$ in the second line of Eq. (31). To illustrate the resonant behaviors of the annihilation, in Fig. 4 we present the solar thermally averaged cross section $\langle (\sigma v_{\text{rel}})_0 \mathcal{S}_0 \rangle_{\odot}$ and $\langle (\sigma_{\phi\phi} v_{\text{rel}})_{\text{tree}} \mathcal{S}_1 \rangle_{\odot}$ as the functions of the DM mass m_{χ} and $z = m_{\chi}/m_{\phi}$, respectively, with coupling $\alpha_{\chi} = 0.01$. The DM mass parameter m_{χ} actually reflects the dependence of the DM velocity within the Sun through the relation $u_0 = \sqrt{2T_{\chi}/m_{\chi}}$ in Eq. (30).

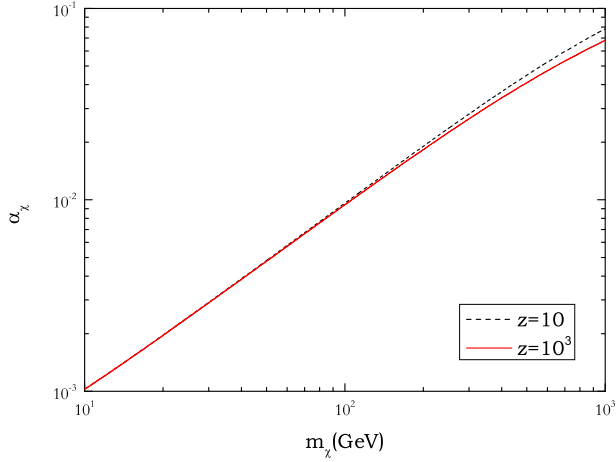


Figure 5: The coupling strength α_χ required to give the correct relic abundance as a function of DM mass m_χ for $z = 10$ (black dashed) and 10^3 (red solid), under the condition $\eta = 10^{-2}$. See text for details.

C. Constraints from the relic density

In Sec.IV A we calculate the self-capture rate C_s with free parameters α_χ and $z = m_\chi/m_\phi$. However, the coupling strength α_χ is not an independent parameter as it can be determined by the observational DM relic density with specific m_χ and m_ϕ . For the given m_χ , m_ϕ and α_χ the evolution of the relic density of DM is described by the following Boltzmann equation [77, 78]

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\text{ann}}v_{\text{rel}}\rangle (n_\chi^2 - n_\chi^{\text{eq}2}), \quad (34)$$

where n_χ and n_χ^{eq} are the number density of the DM and its counterpart in equilibrium with the background particles, respectively, H the Hubble constant and $\langle\sigma_{\text{ann}}v_{\text{rel}}\rangle$ is the thermally averaged cross section introduced in Eq. (31). In practice by changing variables such that $Y = n_\chi/s$ ($Y^{\text{eq}} = n_\chi^{\text{eq}}/s$) with s the entropy density and $x = m_\chi/T$, the above Boltzmann equation can be rewritten as follows [77, 78]

$$\frac{dY}{dx} = -\sqrt{\frac{\pi}{45}}m_{\text{P}}m_\chi \frac{(g_{*s}/\sqrt{g_*})}{x^2} \langle\sigma_{\text{ann}}v_{\text{rel}}\rangle (Y^2 - Y^{\text{eq}2}), \quad (35)$$

where g_{*s} and g_* are the effective degrees of freedom for entropy and energy density, respectively, and the Planck mass $m_{\text{P}} = 1.22 \times 10^{19}$ GeV. We follow Ref. [58, 79] to numerically solve Eq. (35) so as to give the coupling α_χ determined by the relic density. In Fig. 5 shown is the value of α_χ as a function of m_χ and $z = m_\chi/m_\phi$, with a specific value $\eta = 10^{-2}$ introduced in Eq. (31). One can see from Fig. 5 that owing to the high velocity at the chemical decoupling stage, α_χ is insensitive to the Sommerfeld effect. Here we note that the effect of the kinetic decoupling is not included in our calculation, because it is insignificant for the case of p -wave-dominated annihilation when the temperature of kinetic decoupling T_{kd}

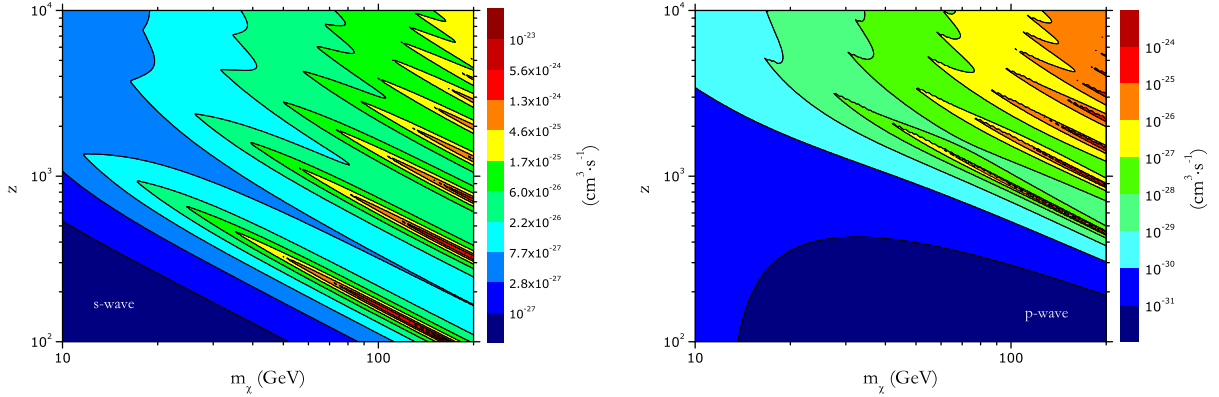


Figure 6: The solar thermal cross section $\langle\sigma_{\text{ann}}v_{\text{rel}}\rangle_{\odot}$ determined by the relic density for the s -wave and p -wave annihilation, respectively. See text for details.

is much less than that of the chemical decoupling T_f , even if the Sommerfeld enhancement is taken into consideration [42, 58, 80]. This requirement is in coincidence with the onset of the Sommerfeld effect: a large $z = m_{\chi}/m_{\phi} \gg 1$ guarantees that $T_f \simeq m_{\chi}/25 \gg m_{\phi} \sim T_{\text{kd}}^{\phi}$, where the kinetic decoupling temperature for the χ - ϕ interaction T_{kd}^{ϕ} provides a maximal value for T_{kd} [12]. Given $\eta = 0.01$, in Fig. 6 we use the relic density to calculate the thermally averaged cross section for the s -wave $\langle(\sigma v_{\text{rel}})_0 \mathcal{S}_0\rangle_{\odot}$ and p -wave annihilation $\langle(\sigma_{\phi\phi} v_{\text{rel}})_{\text{tree}} \mathcal{S}_1\rangle_{\odot}$ within the Sun, respectively. We note that smaller η will not bring remarkable changes to Fig. 6 because the relic density turns out to remain constant as the s -wave component gets smaller ($\eta < 0.01$) [58].

V. CONSEQUENCES ON THE NEUTRINO TELESCOPE

The signal of the solar DM is associated with the observation of primary and secondary high energy neutrinos, the annihilation products of the DM accumulated in the center of the Sun. Among other goals, many terrestrial neutrino detection projects such as Ice-Cube [81], Super-Kamiokande [82], Baikal Neutrino Project [83] and ANTARES [84] are dedicated to such observation, and next generation telescopes including PINGU [85] and Hyper-Kamiokande [86] are also in proposal. In general the neutrino differential flux at the detector location can be schematically described as

$$\frac{d\Phi_{\nu}}{dE_{\nu}} = \frac{\Gamma_A}{4\pi d_{\odot}^2} \frac{dN_{\nu}}{dE_{\nu}}, \quad (36)$$

where Γ_A is the annihilation rate introduced in Eq. (9), d_{\odot} the Sun-Earth distance, and dN_{ν}/dE_{ν} denotes the energy spectrum of neutrino $\nu_{e,\mu,\tau}$ and $\bar{\nu}_{e,\mu,\tau}$ per DM annihilation event, after folding the effects of annihilation branch ratio, hadronization of quarks and neutrino oscillation, which are usually simulated with DarkSUSY [87] and WimpSim [88]. For the purpose of illustration the neutrino flux is connected to the readout at the detector terminal with the following relation:

$$N_\mu = \int_0^{t_{\text{exp}}} dt \int_{\Delta\Omega} d\Omega \int_{E_{\text{th}}} \frac{d\Phi_\nu}{dE_\nu} A_\nu(E_\nu) dE_\nu, \quad (37)$$

where N_μ is the number of the muon events that arise from the charge and neutral current interaction between the incident neutrino and medium surrounding the detector, the effects of which is encoded in the effective area A_ν , E_{th} the threshold energy of the detector, $\Delta\Omega = 2\pi(1 - \cos\Psi)$ being the solid angle surrounding the Sun with angular resolution Ψ , and t_{exp} is the exposure time. If no excess of muon over the background is observed, an upper limit on the annihilation rate Γ_A can be obtained.

Now we use the recent release of the Super-Kamiokande (SK) analysis [57] to constrain the strength of the DM-nucleon interaction in the presence of the Sommerfeld effect, in terms of the upper limits on the nuclear capture rate C_n that depends on the specific DM-nucleon interaction model. One can translate these constraints back on the couplings of the DM-nucleon interaction with the nuclear form factors given by Ref. [89, 90], once the interaction is specified. For the purpose of simple illustration we assume a leptophilic four-fermion interaction in Eq. (3), so that the incoming DM particle with velocity w interacts with the target nucleus through the Higgs portal. The relevant differential cross section is expressed as

$$\frac{d\sigma_{\chi N}}{dq^2} = \alpha_\chi f_N^2 \sin^2\theta \left(\frac{m_p}{v_{\text{EW}}}\right)^2 \frac{A^2 F_N^2(q^2)}{(q^2 + m_\phi^2)^2} \frac{1}{w^2}, \quad (38)$$

where q is the transferred momentum of the incident DM particle, and A is the atomic number of the target nucleus. The nucleon mass is approximated as the protonic one m_p , and the coefficient $f_N = 0.351$ arises from the scalar bilinear for the underlying quarks. $F_N^2(q^2) = [3j_1(qR_1)/(qR_1)]^2 e^{-q^2 s^2}$ is the nuclear form factor, where $j_1(x) = \sin(x)/x^2 - \cos(x)/x$ is the Bessel spherical function of the first kind, and the parameter $R_1 = \sqrt{R_0^2 - 5s^2}$ with $R_0 \simeq 1.23A^{1/3}$ fm, and $s \simeq 1$ fm [91].

Current DM direct detections provide a stringent upper bound on the mixing angle such that $\sin\theta \lesssim 10^{-8}$ [66]. Mixing angle at such scale is 2 to 3 orders of magnitude smaller for the light force-carrier ϕ to decay before $t \sim 1$ s, jeopardizing the success of the BBN, so ϕ is assumed to decay quickly to the sterile neutrino N instead [66], as explained below Eq. (3). As a consequence we regard the s -wave annihilation as the only source relevant for the DM indirect detection. In Fig. 7 we present the upper limits at 90% confidence level (C. L.) on C_n imposed by SK with various $z = m_\chi/m_\phi = 10^2, 10^3$, and 10^4 , and $\eta = 10^{-2}, 10^{-4}, 10^{-5}$ and 10^{-6} , respectively. As is discussed similarly in Ref. [57], two extreme benchmark scenarios in which DM exclusively annihilate via hard ($\tau^+\tau^-$) and soft ($b\bar{b}$) s -wave channel are taken into consideration over a DM mass range from 10 to 100 GeV. The number of the trapped solar DM particles N_χ , or equivalently, the nuclear capture rate C_n is constrained by the upper limit on the annihilation rate Γ_{up} with the following relation:

$$\Gamma_{\text{up}} \geq \frac{1}{2} C_n N_\chi^2 \text{BR}_s, \quad (39)$$

where the branch ratio of the s -wave annihilation which is responsible for the neutrino signal

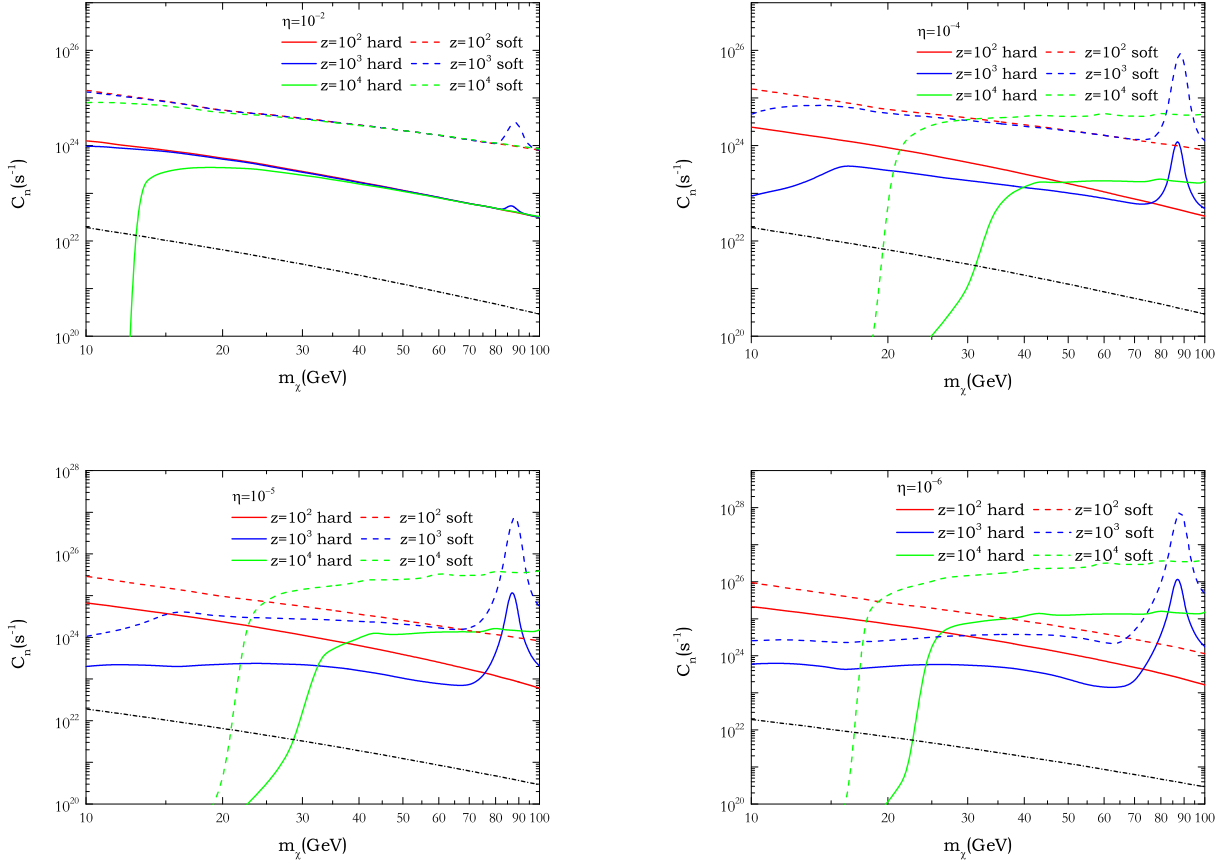


Figure 7: SK 90% C. L. upper limits on nuclear capture rate C_n over a range of DM masses for $\eta = 10^{-2}, 10^{-4}, 10^{-5}$ and 10^{-6} , respectively. Solid (dashed) lines indicate hard (soft) annihilation channels. Shown in the dash-dot line is the nuclear capture rate calculated with parameters $\sin \theta = 10^{-8}$ and $z = 10^4$. See text for details.

can be expressed as

$$\text{BR}_s = \langle \eta (\sigma v_{\text{rel}})_0 \mathcal{S}_0 \rangle_{\odot} / \left[\langle (\sigma_{\phi\phi} v_{\text{rel}})_{\text{tree}} \mathcal{S}_1 \rangle_{\odot} + \langle \eta (\sigma v_{\text{rel}})_0 \mathcal{S}_0 \rangle_{\odot} \right]. \quad (40)$$

The constraints on C_n are affected mainly by three factors: Γ_{up} , z and η . As a rough estimate, Eq. (9) can be used to explain the features of the constraints in Fig. 7: when the Sommerfeld effect is insignificant ($z = 10^2, 10^3$) so that $\Gamma_{\text{up}} \gg C_s^2 / (2C_a)$, upper limits are sensitive to η through the dependence on BR_s ; the sharp increases of the sensitivity on C_n in Fig. 7 imply that $\Gamma_{\text{up}} \sim \mathcal{O}(C_s^2 \text{BR}_s / (2C_a))$ when the Sommerfeld effect turns remarkable ($z = 10^4$), which is of our interest in this study. For reference purpose in Fig. 7 we also present the nuclear capture rate (in the black dash-dot line) derived from the DM-nucleus cross section in Eq. (38), for which the values of the ϕ -Higgs mixing angle $\sin \theta = 10^{-8}$ and $z = 10^4$ are adopted. Parameters at such scales are comparable to the sensitivity of direct detection discussed in Ref. [66]. Although the direct detection approaches are proved to be efficient in exploring the spin-independent nucleon-DM interaction, the solar neutrino

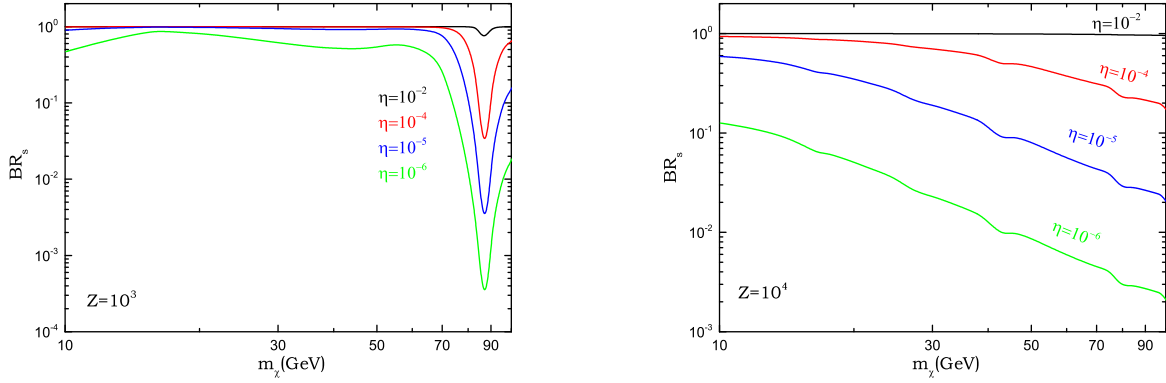


Figure 8: For the given $z = 10^3$ and $z = 10^4$, branch ratio of the s -wave channel as the function of m_χ in the Sun.

detection may impose even more stringent constraints on that as the relevant parameter space is significantly squeezed by the Sommerfeld effect. Such phenomenon occurs at $z = 10^4$ for the present SK sensitivity, as is shown in Fig. 7.

It is also noted that when the s -wave component is sufficiently small that its cross section drops below the p -wave counterpart, constraints will turn loose because that's when BR_s begins to be suppressed by a small η . To get some sense, in Fig. 8 we plot BR_s as a function of DM mass m_χ for $z = 10^3$ and $z = 10^4$. It is evident that the dips in BR_s in the left panel are attributed to the p -wave resonance encountered along the cross section at $z = 10^3$ in Fig. 6, and hence are responsible for the corresponding bumps in sensitivity on C_n in Fig. 7 for $z = 10^3$.

VI. CONCLUSIONS

In this work we investigate an example of the solar DM that self-interact via light force carriers, specifically, Majorana DM particles scattering with long-range self-interaction and annihilating dominantly through p -wave channel. To optimize and cover the relevant phenomenology of the solar SIDM as comprehensively as possible, we study a scenario where a small but non-vanishing quota of DM annihilation proceeded through the s -wave channel in the thermal decoupling epoch, so that the self-capture, both s - and p -wave annihilation are all involved in our consideration.

Instead of evaluating relevant DM self-capture and annihilation processes at tree level, we calculate them in a non-perturbative way by solving the Schrödinger equation with Yukawa potential. We find that the Sommerfeld effect can be significant for DM self-capture if coupling α_χ and mass ratio z are regarded as free parameters. Besides, the impact of the light mediator on the thermal freeze-out that is assumed to produce the the observed DM relic density is also taken into consideration, and is used to determine the self-interaction coupling α_χ for the given masses of DM particle and mediator.

Moreover we also explore the consequence of the Sommerfeld effect on terrestrial neutrino telescopes which is utilized to probe signals from the solar DM. Since the light mediators produced from the p -wave DM annihilation do not produce any observable signal at the terrestrial neutrino telescope, the neutrino signals originate exclusively from the s -wave component, which may become close to or even dominant over its p -wave counterpart within the Sun, due to the Sommerfeld enhancement. Therefore we combine the sensitivity of SK and the observed DM relic density to constrain the nuclear capture rate, or equivalently, the strength of DM-nucleon interaction. Due to SK's high sensitivity, the constraint on the DM-nucleon interaction can be remarkably tightened in the low DM mass region, where the Sommerfeld effect on the DM self-capture rate has not yet been outstripped by the m_χ^{-3} suppression.

It is also noted that while our discussion is based on a generic class of models in which the fermion DM particles self-interact via a light *scalar* mediator, the consequence of this force-carrier in astrophysics and cosmology is another interesting topic that is beyond the scope of this work (For a most recent study, see Ref. [92]). Besides, as a tentative study on the phenomenology of the solar SIDM, our discussion can be further extended to other SIDM models, not confined to the case of scalar mediator or Majorana DM.

Simple as it is, this work has captured the main features of the solar SIDM. To put it in a nutshell, the interplay between the long-range dark force and the solar neutrino detection is encoded in the following expression up to a possible correction due to the self-capture saturation,

$$\Gamma_{\text{up}} \geq \frac{1}{2} \left[C_n + \frac{1}{2} \frac{C_s^2 V_{\text{eff}}}{\left(\langle \sigma v_{\text{rel}} \rangle_{\text{s-wave}} + \langle \sigma v_{\text{rel}} \rangle_{\text{p-wave}} \right)} + \dots \right] \text{BR}_{\text{signal}}, \quad (41)$$

where $\langle \sigma v_{\text{rel}} \rangle_{\text{s-wave}}$ ($\langle \sigma v_{\text{rel}} \rangle_{\text{p-wave}}$) is the s -wave (p -wave) thermal cross section, and $\text{BR}_{\text{signal}}$ is the branch ratio for the signal annihilation channel. Qualitatively speaking, a model with a combination of enhanced C_s and velocity-suppressed $\langle \sigma v_{\text{rel}} \rangle_{\text{p-wave}}$ may maximize the novel effects of the SIDM, as long as such enhancement has not been offset by an extremely small $\text{BR}_{\text{signal}}$. For instance, in the model where ϕ decays to the active neutrinos before the BBN [66], $\text{BR}_{\text{signal}}$ will be significantly boosted if such decay proceeds before ϕ reaches the Earth. In this case, even for a smaller z the neutrino detector SK will lead in the sensitivity on C_n .

Acknowledgments

We would like to thank Yang Ziqing for useful discussion. This work is supported in part by the National Basic Research Program of China (973 Program) under Grants No. 2010CB833000; the National Nature Science Foundation of China (NSFC) under Grants No. 10905084, No. 11335012 and No. 11475237; The numerical calculations were done using the HPC Cluster of SKLTP/ITP-CAS.

Appendix A: Distribution of the self-interacting dark matter in the sun

The distribution of the DM particles trapped in the Sun has been well studied in the literature [93–95]. While in Ref. [93] authors adopts a Maxwellian velocity distribution $f_\chi(\mathbf{v}) \propto \exp(-E_k/T_\chi) \equiv \exp[-m_\chi v^2/(2T_\chi)]$,* to describe the non-thermal equilibrium between the DM particles and solar nuclei, the studies in Ref. [94, 95] indicate a deviation from the Maxwellian form, in a manner that the actual velocity distribution is suppressed at the tail and tends to be anisotropic at large radius. Such deviation can be attributed to the fact that the energetic collisions that send the DM particles into the high orbits occur predominantly near the hotter core of the Sun. Thus one expects a lower angular momentum distribution, or equivalently a larger radial-velocity component (v_r/v) for the high-energy orbits. This is illustrated in Fig. 2 of Ref. [95], where the normalized average kinetic energy of the DM particles $(2/3)\langle E_k \rangle/T_\chi$ decreases toward the outer layers of the Sun, and the average square of the radial velocity component $\langle (v_r/v)^2 \rangle$ deviates from $1/3$ at radii larger than around $0.2 R_\odot$, indicating an anisotropic nature of the velocity distribution beyond that radius. Nevertheless, it should be stressed that while such correction to the thermal distribution is necessary for the evaluation of the energy transport and evaporation in the Sun that rely sensitively on the high end of the velocity distribution [94, 95], the phenomenology of interest in this study such as self-capture and annihilation can still be described with the bulk of the Boltzmann form to a good approximation.

To illustrate this point, we first investigate the solar DM distribution by following the “Brownian motion” method pioneered by the author of Ref. [94]. A total number of 10^5 successive collisions between the 10 GeV trial DM particle and solar elements are simulated. In the left panel of Fig. 9 we present the quantity $(2/3)\langle E_k \rangle/T_\chi$ and $\langle (v_r/v)^2 \rangle$ from the solar center up to $0.1 R_\odot$, a radius sufficient to envelope almost all the DM particles trapped in the Sun (see Eq. (15)). The curves would be horizontal (at 1 and $1/3$ respectively) for a thermal distribution. We compare the simulated velocity distribution with the Maxwellian form in the right panel, where the velocity variable v is nondimensionalized in terms of the velocity unit $(GM_\odot/R_\odot)^{1/2} \approx 436 \text{ km} \cdot \text{s}^{-1}$. It is evident that the Maxwellian distribution with the characteristic temperature T_χ indeed provides a good description of the realistic solar DM distribution for the mass of interest. $T_\chi = 0.967 T_\odot(0)$ is determined from Eq. (13). For larger DM masses, one can approximate T_χ as the solar center temperature $T_\odot(0)$.

In addition, the Boltzmann form is expected to remain an accurate description for the solar SIDM because it naturally describes the thermal distribution of the SIDM particles gathering in the solar core, while the effective temperature T_χ accounts for the non-thermal equilibrium between the SIDM particles and the solar nuclei to a good approximation. Therefore we still adopt the Boltzmann distribution in the evaluation of self-capture and annihilation in the Sun.

* By demand of satisfying the Boltzmann equation, the Maxwellian velocity distribution in turn leads to the Boltzmann distribution $f_\chi(\mathbf{r}, \mathbf{v}) \propto \exp[-(V(r) + E_k)/T_\chi]$, and hence the density distribution $n_\chi(\mathbf{r}) \propto \exp[-V(r)/T_\chi]$, with $V(r)$ the gravitational potential of the DM particle at radius r .

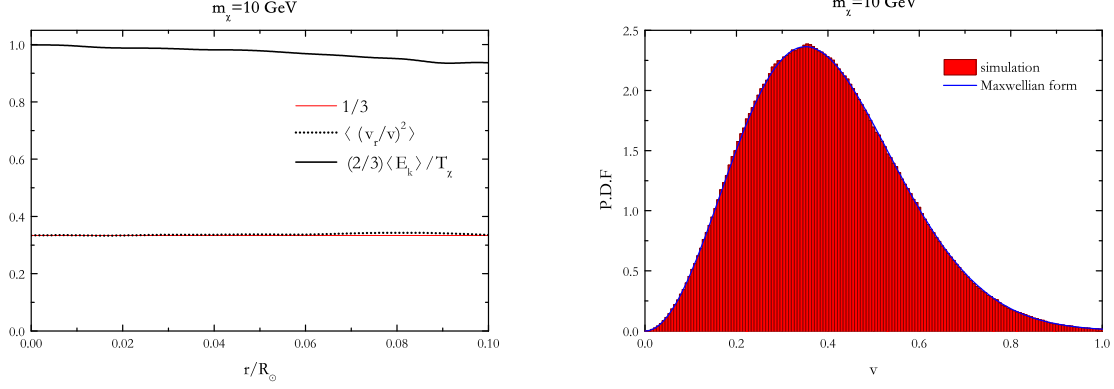


Figure 9: **Left panel:** the normalized average kinetic energy $(2/3) \langle E_k \rangle / T_\chi$ (solid) and the average square of the radial velocity component $\langle (v_r/v)^2 \rangle$ (dotted) for DM mass $m_\chi = 10 \text{ GeV}$. **Right panel:** The integrated probability density function (P.D.F) of the solar DM particles obtained from the Brownian motion simulation (red histogram). For comparison also shown is the Maxwellian distribution (blue lines). See text for details.

Appendix B: Self-evaporation rate of the trapped dark matter

In this appendix we will verify the assumption made below Eq. (5) that the evaporation rate due to the self-interaction of dark matter is negligible compared to the annihilation rate. To this end we try to estimate the dark matter self-evaporation rate in the approach proposed in Ref. [68], by which we start with the self-evaporation rate ϵ_s in a unit volume :

$$\epsilon_s = \frac{n_\chi^2}{2} \int d^3 w_1 f_\chi(\mathbf{w}_1) \int d^3 w_2 f_\chi(\mathbf{w}_2) |\mathbf{w}_1 - \mathbf{w}_2| \int_{v_{\text{esc}}} \frac{d\sigma(\mathbf{w}_1, \mathbf{w}_2; \mathbf{v}_1, \mathbf{v}_2)}{dv_1 dv_2} d^3 v_1 d^3 v_2, \quad (\text{B1})$$

where \mathbf{w}_1 and \mathbf{w}_2 are respectively the initial velocities of the trapped DM before scattering, with their corresponding Maxwellian distributions $f_\chi(\mathbf{w}_1)$ and $f_\chi(\mathbf{w}_2)$ introduced in Eq. (30), \mathbf{v}_1 and \mathbf{v}_2 being the final velocity after collision, one of which must exceed the local escape velocity v_{esc} in the Sun so as to be evaporated, and $d\sigma(\mathbf{w}_1, \mathbf{w}_2; \mathbf{v}_1, \mathbf{v}_2) / dv_1 dv_2$ schematically represents relevant differential cross section. Then the conservation of energy and momentum implies

$$\begin{aligned} f_\chi(\mathbf{w}_1) f_\chi(\mathbf{w}_2) &= f_\chi(\mathbf{v}_1) f_\chi(\mathbf{v}_2), \\ |\mathbf{w}_1 - \mathbf{w}_2| &= |\mathbf{v}_1 - \mathbf{v}_2|, \end{aligned} \quad (\text{B2})$$

so we have

$$\begin{aligned} \epsilon_s &= \frac{n_\chi^2}{2} \int d^3 v_1 f_\chi(\mathbf{v}_1) \int d^3 v_2 f_\chi(\mathbf{v}_2) |\mathbf{v}_1 - \mathbf{v}_2| \int \frac{d\sigma(\mathbf{v}_1, \mathbf{v}_2; \mathbf{w}_1, \mathbf{w}_2)}{dw_1 dw_2} d^3 w_1 d^3 w_2 \\ &\simeq n_\chi^2 \int_{v > v_{\text{esc}}} d^3 v f_\chi(\mathbf{v}) v \int \frac{d\sigma(\mathbf{v}, 0; \mathbf{w}_1, \mathbf{w}_2)}{dw_1 dw_2} d^3 w_1 d^3 w_2 \\ &= n_\chi^2 \int_{v > v_{\text{esc}}} d^3 v f_\chi(\mathbf{v}) v \sigma_{\text{sc}}(v), \end{aligned} \quad (\text{B3})$$

where we make approximation $|\mathbf{v}_1 - \mathbf{v}_2| \simeq |\mathbf{v}_1| = v_1$ for the case $v_1 > v_{\text{esc}} \gg v_2$, in which the contribution to evaporation rate doesn't suffer the Maxwellian tail suppression of $f_\chi(\mathbf{v}_2)$, and $\sigma_{\text{sc}}(v)$ is no other than the self-capture cross section with incident velocity v . On the other hand, the annihilation rate in a unit volume is written as

$$\begin{aligned} n_\chi^2 \langle \sigma_{\text{ann}} v \rangle &= n_\chi^2 \int d^3v_1 f_\chi(\mathbf{v}_1) \int d^3v_2 f_\chi(\mathbf{v}_2) |\mathbf{v}_1 - \mathbf{v}_2| \sigma_{\text{ann}}(|\mathbf{v}_1 - \mathbf{v}_2|) \\ &\simeq n_\chi^2 \int d^3v \left(v/\sqrt{2} \right) f_\chi(\mathbf{v}/\sqrt{2}) v \sigma_{\text{ann}}(v) \\ &= n_\chi^2 \int d^3v f_\chi(\mathbf{v}) \sqrt{2} v \sigma_{\text{ann}}(\sqrt{2} v), \end{aligned} \quad (\text{B4})$$

where $\sigma_{\text{ann}}(v)$ is the annihilation cross section with relative velocity v . Given that compared with Eq. (B4) the integral over subspace $\int_{v>v_{\text{esc}}} d^3v f_\chi(\mathbf{v})$ in Eq. (B3) so overwhelmingly suppresses ϵ_s in the DM mass region of our interest that we can safely ignore the contribution from dark matter self-interaction in this study. Finally it is worth mentioning that while our discussion is based on the assumption of a large mean free path for the SIDM particles, the self-evaporation will be heavily suppressed if the optical depth is much smaller than the DM radius. Because in such case, the would-be evaporation events will end up being blocked by multiple collisions, which further favors our conclusion.

-
- [1] E. D. Carlson, M. E. Machacek, and L. J. Hall, *Self-interacting dark matter*, *Astrophys. J.* **398** (Oct., 1992) 43–52.
 - [2] A. A. de Laix, R. J. Scherrer, and R. K. Schaefer, *Constraints on Self-interacting Dark Matter*, *Astrophys. J.* **452** (Oct., 1995) 495, [astro-ph/9502087].
 - [3] C. J. Hogan and J. J. Dalcanton, *New dark matter physics: clues from halo structure*, *Phys.Rev.* **D62** (2000) 063511, [astro-ph/0002330].
 - [4] R. Davé, D. N. Spergel, P. J. Steinhardt, and B. D. Wandelt, *Halo Properties in Cosmological Simulations of Self-interacting Cold Dark Matter*, *Astrophys. J.* **547** (Feb., 2001) 574–589, [astro-ph/0006218].
 - [5] J. Hisano, S. Matsumoto, and M. M. Nojiri, *Explosive Dark Matter Annihilation*, *Physical Review Letters* **92** (Jan., 2004) 031303, [hep-ph/0307216].
 - [6] J. Hisano, S. Matsumoto, M. M. Nojiri, and O. Saito, *Nonperturbative effect on dark matter annihilation and gamma ray signature from the galactic center*, *Phys. Rev. D* **71** (Mar., 2005) 063528, [hep-ph/0412403].
 - [7] A. Sommerfeld *Annalen der Physik* **403** (1931) 257.
 - [8] J. Hisano, S. Matsumoto, and M. M. Nojiri, *Unitarity and higher order corrections in neutralino dark matter annihilation into two photons*, *Phys.Rev.* **D67** (2003) 075014, [hep-ph/0212022].
 - [9] J. Hisano, S. Matsumoto, and M. M. Nojiri, *Explosive dark matter annihilation*, *Phys.Rev.Lett.* **92** (2004) 031303, [hep-ph/0307216].

- [10] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, and N. Weiner, *A Theory of Dark Matter*, *Phys.Rev.* **D79** (2009) 015014, [arXiv:0810.0713].
- [11] J. L. Feng, M. Kaplinghat, and H.-B. Yu, *Halo Shape and Relic Density Exclusions of Sommerfeld-Enhanced Dark Matter Explanations of Cosmic Ray Excesses*, *Phys.Rev.Lett.* **104** (2010) 151301, [arXiv:0911.0422].
- [12] J. L. Feng, M. Kaplinghat, and H.-B. Yu, *Sommerfeld Enhancements for Thermal Relic Dark Matter*, *Phys.Rev.* **D82** (2010) 083525, [arXiv:1005.4678].
- [13] **PAMELA** , O. Adriani et al., *An anomalous positron abundance in cosmic rays with energies 1.5-100 GeV*, *Nature* **458** (2009) 607–609, [arXiv:0810.4995].
- [14] O. Adriani, G. Barbarino, G. Bazilevskaya, R. Bellotti, M. Boezio, et al., *A statistical procedure for the identification of positrons in the PAMELA experiment*, *Astropart.Phys.* **34** (2010) 1–11, [arXiv:1001.3522].
- [15] **Fermi-LAT** , M. Ackermann et al., *Measurement of separate cosmic-ray electron and positron spectra with the Fermi Large Area Telescope*, *Phys.Rev.Lett.* **108** (2012) 011103, [arXiv:1109.0521].
- [16] **AMS** , L. Accardo et al., *High Statistics Measurement of the Positron Fraction in Primary Cosmic Rays of 0.5–500 GeV with the Alpha Magnetic Spectrometer on the International Space Station*, *Phys. Rev. Lett.* **113** (2014) 121101.
- [17] J. Kopp, *Constraints on dark matter annihilation from AMS-02 results*, *Phys.Rev.* **D88** (2013) 076013, [arXiv:1304.1184].
- [18] Q. Yuan, X.-J. Bi, G.-M. Chen, Y.-Q. Guo, S.-J. Lin, et al., *Implications of the AMS-02 positron fraction in cosmic rays*, *Astropart.Phys.* **60** (2015) 1–12, [arXiv:1304.1482].
- [19] I. Cholis and D. Hooper, *Dark Matter and Pulsar Origins of the Rising Cosmic Ray Positron Fraction in Light of New Data From AMS*, *Phys.Rev.* **D88** (2013) 023013, [arXiv:1304.1840].
- [20] H.-B. Jin, Y.-L. Wu, and Y.-F. Zhou, *Implications of the first AMS-02 measurement for dark matter annihilation and decay*, *JCAP* **1311** (2013) 026, [arXiv:1304.1997].
- [21] Z.-P. Liu, Y.-L. Wu, and Y.-F. Zhou, *Sommerfeld enhancements with vector, scalar and pseudoscalar force-carriers*, *Phys.Rev.* **D88** (2013) 096008, [arXiv:1305.5438].
- [22] A. Ibarra, A. S. Lamperstorfer, and J. Silk, *Dark matter annihilations and decays after the AMS-02 positron measurements*, *Phys.Rev.* **D89** (2014), no. 6 063539, [arXiv:1309.2570].
- [23] H.-B. Jin, Y.-L. Wu, and Y.-F. Zhou, *Cosmic ray propagation and dark matter in light of the latest AMS-02 data*, arXiv:1410.0171.
- [24] G. Giesen, M. Boudaud, Y. Genolini, V. Poulin, M. Cirelli, et al., *AMS-02 antiprotons, at last! Secondary astrophysical component and immediate implications for Dark Matter*, arXiv:1504.04276.
- [25] H.-B. Jin, Y.-L. Wu, and Y.-F. Zhou, *Upper limits on DM annihilation cross sections from the first AMS-02 antiproton data*, arXiv:1504.04604.
- [26] K. Kohri, K. Ioka, Y. Fujita, and R. Yamazaki, *Can we explain AMS-02 antiproton and positron excesses simultaneously by nearby supernovae without pulsars nor dark matter?*, arXiv:1505.01236.
- [27] Y.-H. Chen, K. Cheung, and P.-Y. Tseng, *Dark Matter with multi-annihilation channels and AMS-02 positron excess and antiproton*, arXiv:1505.00134.

- [28] C.-H. Chen, C.-W. Chiang, and T. Nomura, *Dark matter for excess of AMS-02 positrons and antiprotons*, arXiv:1504.07848.
- [29] S.-J. Lin, X.-J. Bi, P.-F. Yin, and Z.-H. Yu, *Implications for dark matter annihilation from the AMS-02 \bar{p}/p ratio*, arXiv:1504.07230.
- [30] K. Hamaguchi, T. Moroi, and K. Nakayama, *AMS-02 Antiprotons from Annihilating or Decaying Dark Matter*, arXiv:1504.05937.
- [31] M. Ibe, S. Matsumoto, S. Shirai, and T. T. Yanagida, *Wino Dark Matter in the light of AMS-02 2015*, arXiv:1504.05554.
- [32] M. Lattanzi and J. I. Silk, *Can the WIMP annihilation boost factor be boosted by the Sommerfeld enhancement?*, *Phys.Rev.* **D79** (2009) 083523, [arXiv:0812.0360].
- [33] J. L. Feng, M. Kaplinghat, H. Tu, and H.-B. Yu, *Hidden Charged Dark Matter*, *JCAP* **0907** (2009) 004, [arXiv:0905.3039].
- [34] T. R. Slatyer, *The Sommerfeld enhancement for dark matter with an excited state*, *JCAP* **1002** (2010) 028, [arXiv:0910.5713].
- [35] M. R. Buckley and P. J. Fox, *Dark Matter Self-Interactions and Light Force Carriers*, *Phys.Rev.* **D81** (2010) 083522, [arXiv:0911.3898].
- [36] J. B. Dent, S. Dutta, and R. J. Scherrer, *Thermal Relic Abundances of Particles with Velocity-Dependent Interactions*, *Phys.Lett.* **B687** (2010) 275–279, [arXiv:0909.4128].
- [37] J. Zavala, M. Vogelsberger, and S. D. White, *Relic density and CMB constraints on dark matter annihilation with Sommerfeld enhancement*, *Phys.Rev.* **D81** (2010) 083502, [arXiv:0910.5221].
- [38] S. Hannestad and T. Tram, *Sommerfeld Enhancement of DM Annihilation: Resonance Structure, Freeze-Out and CMB Spectral Bound*, *JCAP* **1101** (2011) 016, [arXiv:1008.1511].
- [39] J. Hisano, M. Kawasaki, K. Kohri, T. Moroi, K. Nakayama, et al., *Cosmological constraints on dark matter models with velocity-dependent annihilation cross section*, *Phys.Rev.* **D83** (2011) 123511, [arXiv:1102.4658].
- [40] D. Hooper, N. Weiner, and W. Xue, *Dark Forces and Light Dark Matter*, *Phys.Rev.* **D86** (2012) 056009, [arXiv:1206.2929].
- [41] B. Bellazzini, M. Cliche, and P. Tanedo, *Effective theory of self-interacting dark matter*, *Phys.Rev.* **D88** (2013), no. 8 083506, [arXiv:1307.1129].
- [42] S. Tulin, H.-B. Yu, and K. M. Zurek, *Beyond Collisionless Dark Matter: Particle Physics Dynamics for Dark Matter Halo Structure*, *Phys.Rev.* **D87** (2013), no. 11 115007, [arXiv:1302.3898].
- [43] R. Laha and E. Braaten, *Direct detection of dark matter in universal bound states*, *Phys. Rev.* **D89** (2014), no. 10 103510, [arXiv:1311.6386].
- [44] E. Gabrielli and M. Raidal, *Exponentially spread dynamical Yukawa couplings from nonperturbative chiral symmetry breaking in the dark sector*, *Phys.Rev.* **D89** (2014), no. 1 015008, [arXiv:1310.1090].
- [45] R. Foot and S. Vagnozzi, *Diurnal modulation signal from dissipative hidden sector dark matter*, arXiv:1412.0762.
- [46] R. Foot and S. Vagnozzi, *Dissipative hidden sector dark matter*, *Phys.Rev.* **D91** (2015) 023512, [arXiv:1409.7174].

- [47] I. Lopes, P. Panci, and J. Silk, *Helioseismology with long range dark matter-baryon interactions*, *Astrophys.J.* **795** (2014) 162, [arXiv:1402.0682].
- [48] R. Laha, *Directional detection of dark matter in universal bound states*, arXiv:1505.02772.
- [49] M. Blennow and S. Clementz, *Asymmetric capture of Dirac dark matter by the Sun*, arXiv:1504.05813.
- [50] J. Fan, A. Katz, L. Randall, and M. Reece, *Double-Disk Dark Matter*, *Phys.Dark Univ.* **2** (2013) 139–156, [arXiv:1303.1521].
- [51] B. Robertson and A. Zentner, *Dark Matter Annihilation Rates with Velocity-Dependent Annihilation Cross Sections*, *Phys.Rev.* **D79** (2009) 083525, [arXiv:0902.0362].
- [52] A. R. Zentner, *High-energy neutrinos from dark matter particle self-capture within the Sun*, *Phys. Rev. D* **80** (Sept., 2009) 063501, [arXiv:0907.3448].
- [53] I. F. Albuquerque, C. Perez de Los Heros, and D. S. Robertson, *Constraints on self interacting dark matter from IceCube results*, *JCAP* **1402** (2014) 047, [arXiv:1312.0797].
- [54] C.-S. Chen, F.-F. Lee, G.-L. Lin, and Y.-H. Lin, *Probing Dark Matter Self-Interaction in the Sun with IceCube-PINGU*, *JCAP* **1410** (2014), no. 10 049, [arXiv:1408.5471].
- [55] J. Fan, A. Katz, and J. Shelton, *Direct and indirect detection of dissipative dark matter*, *JCAP* **1406** (2014) 059, [arXiv:1312.1336].
- [56] C.-S. Chen, G.-L. Lin, and Y.-H. Lin, *Complementary Test of the Dark Matter Self-Interaction by Direct and Indirect Detections*, arXiv:1505.03781.
- [57] **Super-Kamiokande** , K. Choi et al., *Search for neutrinos from annihilation of captured low-mass dark matter particles in the Sun by Super-Kamiokande*, *Phys.Rev.Lett.* **114** (2015), no. 14 141301, [arXiv:1503.04858].
- [58] J. Chen and Y.-F. Zhou, *The 130 GeV gamma-ray line and Sommerfeld enhancements*, *JCAP* **1304** (2013) 017, [arXiv:1301.5778].
- [59] C. P. Burgess, M. Pospelov, and T. ter Veldhuis, *The Minimal model of nonbaryonic dark matter: A Singlet scalar*, *Nucl. Phys.* **B619** (2001) 709–728, [hep-ph/0011335].
- [60] B. Patt and F. Wilczek, *Higgs-field portal into hidden sectors*, hep-ph/0605188.
- [61] S. Andreas, T. Hambye, and M. H. G. Tytgat, *WIMP dark matter, Higgs exchange and DAMA*, *JCAP* **0810** (2008) 034, [arXiv:0808.0255].
- [62] S. Andreas, C. Arina, T. Hambye, F.-S. Ling, and M. H. G. Tytgat, *A light scalar WIMP through the Higgs portal and CoGeNT*, *Phys. Rev.* **D82** (2010) 043522, [arXiv:1003.2595].
- [63] A. Djouadi, O. Lebedev, Y. Mambrini, and J. Quevillon, *Implications of LHC searches for Higgs–portal dark matter*, *Phys. Lett.* **B709** (2012) 65–69, [arXiv:1112.3299].
- [64] M. Pospelov and A. Ritz, *Higgs decays to dark matter: beyond the minimal model*, *Phys. Rev.* **D84** (2011) 113001, [arXiv:1109.4872].
- [65] A. Greljo, J. Julio, J. F. Kamenik, C. Smith, and J. Zupan, *Constraining Higgs mediated dark matter interactions*, *JHEP* **11** (2013) 190, [arXiv:1309.3561].
- [66] C. Kouvaris, I. M. Shoemaker, and K. Tuominen, *Self-Interacting Dark Matter through the Higgs Portal*, *Phys.Rev.* **D91** (2015), no. 4 043519, [arXiv:1411.3730].
- [67] M. Kaplinghat, S. Tulin, and H.-B. Yu, *Direct Detection Portals for Self-interacting Dark Matter*, *Phys.Rev.* **D89** (2014), no. 3 035009, [arXiv:1310.7945].
- [68] K. Griest and D. Seckel, *Cosmic Asymmetry, Neutrinos and the Sun*, *Nucl.Phys.* **B283**

- (1987) 681.
- [69] A. Gould, *Resonant Enhancements in WIMP Capture by the Earth*, *Astrophys.J.* **321** (1987) 571.
 - [70] G. Busoni, A. De Simone, and W.-C. Huang, *On the Minimum Dark Matter Mass Testable by Neutrinos from the Sun*, *JCAP* **1307** (2013) 010, [arXiv:1305.1817].
 - [71] C. Kouvaris, *Probing Light Dark Matter via Evaporation from the Sun*, *Phys. Rev.* **D92** (2015), no. 7 075001, [arXiv:1506.04316].
 - [72] A. Gould, *Cosmological density of WIMPs from solar and terrestrial annihilations*, *Astrophys.J.* **388** (1992) 338–344.
 - [73] A. Serenelli, S. Basu, J. W. Ferguson, and M. Asplund, *New Solar Composition: The Problem With Solar Models Revisited*, *Astrophys.J.* **705** (2009) L123–L127, [arXiv:0909.2668].
 - [74] J. Kumar, J. G. Learned, S. Smith, and K. Richardson, *Tools for Studying Low-Mass Dark Matter at Neutrino Detectors*, *Phys.Rev.* **D86** (2012) 073002, [arXiv:1204.5120].
 - [75] S. Cassel, *Sommerfeld factor for arbitrary partial wave processes*, *J.Phys.* **G37** (2010) 105009, [arXiv:0903.5307].
 - [76] R. Iengo, *Sommerfeld enhancement: General results from field theory diagrams*, *JHEP* **0905** (2009) 024, [arXiv:0902.0688].
 - [77] P. Gondolo and G. Gelmini, *Cosmic abundances of stable particles: Improved analysis*, *Nucl.Phys.* **B360** (1991) 145–179.
 - [78] E. W. Kolb and M. S. Turner, *The Early Universe*, *Front.Phys.* **69** (1990) 1–547.
 - [79] S. Hannestad and T. Tram, *Sommerfeld enhancement of DM annihilation: resonance structure, freeze-out and CMB spectral bound*, *JCAP* **1** (Jan., 2011) 16, [arXiv:1008.1511].
 - [80] J. B. Dent, S. Dutta, and R. J. Scherrer, *Thermal relic abundances of particles with velocity-dependent interactions*, *Physics Letters B* **687** (Apr., 2010) 275–279, [arXiv:0909.4128].
 - [81] IceCube collaboration, M. G. Aartsen, R. Abbasi, Y. Abdou, M. Ackermann, J. Adams, J. A. Aguilar, M. Ahlers, D. Altmann, K. Andeen, and et al., *Search for dark matter annihilations in the Sun with the 79-string IceCube detector*, *ArXiv e-prints* (Dec., 2012) [arXiv:1212.4097].
 - [82] **Super-Kamiokande Collaboration** , T. Tanaka et al., *An Indirect Search for Weakly Interacting Massive Particles in the Sun Using 3109.6 Days of Upward-going Muons in Super-Kamiokande*, *Astrophys. J.* **742** (Dec., 2011) 78, [arXiv:1108.3384].
 - [83] **Baikal Collaboration** , A. Avrorin et al., *Search for neutrino emission from relic dark matter in the Sun with the Baikal NT200 detector*, arXiv:1405.3551.
 - [84] **ANTARES Collaboration** , S. Adrian-Martinez et al., *First results on dark matter annihilation in the Sun using the ANTARES neutrino telescope*, *JCAP* **1311** (2013) 032, [arXiv:1302.6516].
 - [85] **IceCube-PINGU Collaboration** , M. Aartsen et al., *Letter of Intent: The Precision IceCube Next Generation Upgrade (PINGU)*, arXiv:1401.2046.
 - [86] K. Abe, T. Abe, H. Aihara, Y. Fukuda, Y. Hayato, et al., *Letter of Intent: The Hyper-Kamiokande Experiment — Detector Design and Physics Potential —*, arXiv:1109.3262.
 - [87] P. Gondolo, J. Edsjo, P. Ullio, L. Bergstrom, M. Schelke, et al., *DarkSUSY: Computing*

- supersymmetric dark matter properties numerically*, *JCAP* **0407** (2004) 008, [astro-ph/0406204].
- [88] M. Blennow, J. Edsjo, and T. Ohlsson, *Neutrinos from WIMP annihilations using a full three-flavor Monte Carlo*, *JCAP* **0801** (2008) 021, [arXiv:0709.3898].
 - [89] R. Catena and B. Schwabe, *Form factors for dark matter capture by the Sun in effective theories*, arXiv:1501.03729.
 - [90] R. Catena, *Dark matter signals at neutrino telescopes in effective theories*, arXiv:1503.04109.
 - [91] J. D. Lewin and P. F. Smith, *Review of mathematics, numerical factors, and corrections for dark matter experiments based on elastic nuclear recoil*, *Astropart. Phys.* **6** (1996) 87–112.
 - [92] K. Kainulainen, K. Tuominen, and V. Vaskonen, *Self-interacting dark matter and cosmology of a light scalar mediator*, arXiv:1507.04931.
 - [93] D. N. Spergel and W. H. Press, *Effect of hypothetical, weakly interacting, massive particles on energy transport in the solar interior*, *Astrophys. J.* **294** (1985) 663–673.
 - [94] M. Nauenberg, *Energy Transport and Evaporation of Weakly Interacting Particles in the Sun*, *Phys. Rev.* **D36** (1987) 1080.
 - [95] A. Gould, *WIMP Distribution in and Evaporation From the Sun*, *Astrophys. J.* **321** (1987) 560.